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Search Profiling with Partial Knowledge of Deterrence

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Abstract

Though economists engaged in normative study of public policy generally assume that the relevant social planner knows how policy affects population behavior, this rarely is the case in practice. Fundamental identification problems and practical problems of statistical inference make it difficult to learn how policy affects behavior. Hence, there is much reason to consider policy formation when a planner has only partial knowledge of policy impacts.

In this paper, I examine a specific and recently debated aspect of law enforcement—the choice of a *profiling* policy. My concern is not so much to understand the use of personal attributes such as race in profiling policies (though some of my analysis does have implications for detecting discrimination), but rather to understand how a social planner might reasonably choose a profiling policy when he or she only has partial knowledge of how policy affects criminal behavior. I consider both *ex ante* search, which apprehends offenders before their offenses cause social harm, and *ex post* search, which apprehends offenders after completion of their offenses. This paper shows how a social planner having partial knowledge of population offense behavior can "reasonably" choose a search profiling policy.

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1. Introduction

Economists engaged in normative study of public policy generally assume that the relevant social planner knows how policy affects population behavior. For example, economists studying optimal income taxation assume that the planner knows how the tax schedule affects labor supply (e.g., Mirrlees, 1971). Those studying optimal criminal justice systems assume that the planner knows how policing and sanctions affect offense rates (e.g., Polinsky and Shavell, 2000).

In these and other policy domains, social planners may not possess the knowledge that economists assume them to have. Fundamental identification problems and practical problems of statistical inference make it difficult to learn how policy affects behavior. Hence, there is reason to consider policy formation when a planner has only partial knowledge of policy impacts.

I have previously studied an abstract planning problem when policy impacts are partially identified (Manski, 2000; 2002; 2005a, Chapter 2) or are observed in finite samples (Manski, 2004; 2005a, Chapter 3). Here, I examine a specific aspect of law enforcement that has recently been the subject of debate. This is the choice of a *profiling* policy wherein decisions to search for evidence of crime may vary with observable covariates of the persons at risk of being searched. Policies that make search rates vary with personal attributes are variously defended as essential to effective law enforcement and denounced as unfair to classes of persons subjected to relatively high search rates. Variation of search rates by race has been particularly controversial; see, for example, Knowles, Persico, and Todd (2001), Persico (2002), and Dominitz (2003). Whereas recent research on profiling has sought to define and detect racial discrimination, my concern is to understand how a social planner might reasonably choose a profiling policy.

Section 2 poses a planning problem whose objective is to minimize the utilitarian social cost of crime and search. Search is costly per se, and search that reveals a crime entails costs for punishment of offenders. Search is beneficial to the extent that it deters or prevents crime. Deterrence is expressed through the *offense function*, which describes how the offense rate of persons with given covariates varies with the search rate applied to these persons. Prevention occurs when search prevents an offense from causing social harm. Drawing on Manski (2005b), I use the analytically simple case of linear deterrence to illustrate how the optimal search rate may depend on the cost magnitudes and offense function.

Section 3 examines the planning problem when the planner has only partial knowledge of the offense function and, hence, is unable to determine what policy is optimal. To demonstrate general ideas, I consider in depth a specific informational setting that may sometimes be realistic. I suppose that the planner observes the offense rates of a study population whose search rule has previously been chosen. He knows (or finds it credible to assume) that the study population and the population of interest have the same offense function. He also knows that search weakly deters crime; that is, the offense rate weakly decreases as the search rate increases. However, the planner does not know the magnitude of the deterrent effect of search.

In this setting, I first show how the planner can eliminate dominated search rules, ones which are inferior whatever the actual offense function may be. Broadly speaking, low (high) search rates are dominated when the cost of search is low (high); Lemmas 1 and 2 make this precise. I then show how the planner can use the minimax or minimax-regret criterion to choose an undominated search rule. Both criteria are reasonable and tractable, but they yield different policies; Lemmas 3 and 4 derive their explicit forms.

Sections 2 and 3 consider *ex ante* search, which apprehends offenders before their offenses cause social harm. Section 4 performs parallel analysis for *ex post* search, which apprehends offenders after completion of their offenses. Whereas ex ante search both deters and prevents crime, ex post search only deters. The two types of search have different implications for profiling policy. Lemmas 5 through 8 formalize the analysis of ex post search.

Section 5 discusses some variations on the planning problems examined in Sections 2 through 4. Formal study of these variations would require fresh analysis. However, the general idea that a planner with partial knowledge of deterrence can eliminate dominated search rules and select a minimax or minimax-regret rule remains applicable.

Although detection of discrimination is not my direct concern, the analysis in this paper does have implications for that inferential problem. The models studied in Knowles, Persico, and Todd (2001) and in Persico (2002) imply that, in the absence of discrimination, optimal profiling must equalize the offense rates of persons with different covariates, provided that such persons are searched at all. The present model differs from theirs, and it does not produce their conclusion. Perhaps the most important difference is in the objective functions assumed for the agencies that make profiling policy. They assumed that police on the street aim to maximize the probability of successful searches minus the cost of performing searches. I assume that a planner wants to minimize a social cost function with three components: (a) the harm caused by completed offenses, (b) the cost of punishing offenders who are apprehended, and (c) the cost of performing searches.

2. Optimal Ex-Ante Profiling

Let there exist a large population of potential offenders—formally, the population is an uncountable probability space (J, Ω, P) with $P(j) = 0$, $j \in J$. Each member of this population decides whether or not to commit an offense, taking into account the chance that he will be searched. Let $t \in [0, 1]$ denote the probability with which a person is searched. Let $y_j(t) = 1$ if person j chooses to commit an offense when the search probability is t, with $y_j(t) = 0$ otherwise.

The planning problem is to choose the probabilities with which persons are searched. Let person j have observable *fixed* covariates $x_j \in X$, with X being the space of possible covariate values. It is important to my analysis that the planner use only fixed covariates to determine search rates. If search rates vary with *malleable* covariates, persons may choose to manipulate their covariate values so as to lower the probability of search. I permit no such manipulation of covariates.

I assume that it is legal to search differentially among persons with different values of x—if not, then redefine x to be those fixed covariates that the planner can observe and legally use. The planner can a priori distinguish persons with different observed covariates, but cannot distinguish among persons with the same covariates. Hence, a feasible search rule is a function $z: X \rightarrow [0, 1]$ that assigns a homogeneous search rate to all persons with the same value of x, but possibly different search rates to persons with different covariates. I assume that offenders are always apprehended through search but are not apprehended otherwise. The analysis here and in Section 3 assumes that search is ex ante.

Let $p(t, x) \equiv P[y(t) = 1 | x]$ be the offense function, giving the fraction of persons with covariates x who commit an offense when their search rate is t. Under search rule z, the offense rate among persons with covariates x is $p[z(x), x] = P\{y[z(x)] = 1 | x\}$. I need not specify a particular model of offense behavior, but one may find it helpful to envision a threshold-crossing model in which $y_j(t) = 1$ if $t < \tau_j$ and $y_j(t) = 0$ if $t > \tau_j$, where τ_j is a person-specific threshold. Then $P(t < \tau | x) \le p(t, x) \le P(t \le \tau | x)$.

The planner wants to minimize a social cost function with three additive components. These are (a) the harm caused by completed offenses, (b) the cost of punishing offenders who are apprehended, and (c) the cost of performing searches. I assume for simplicity that decisions to commit offenses are statistically independent of the harm that these offenses would cause. Then the social cost of search rule z is

$$(1) \quad S(z) = \int a(x)\cdot p[z(x), x]\cdot[1 - z(x)]dP(x) + \int b(x)\cdot p[z(x), x]\cdot z(x)dP(x) + \int c(x)\cdot z(x)dP(x).$$

Consider the first term on the right hand side. For each $x \in X$, $p[z(x), x]$ is the probability that a person with covariates x commits an offense and $1 - z(x)$ is the probability that such a person is not searched; hence, the product $p[z(x), x]\cdot[1 - z(x)]$ is the probability that a person with covariates x commits an offense that causes social harm. The positive constant a(x) is the mean magnitude of the harm caused by an offense. Integrating across the covariate distribution P(x) yields the aggregate social cost due to harm caused by completed offenses.

Next consider the second term. The product $p[z(x), x]\cdot z(x)$ is the probability that a person with covariates x commits an offense but is apprehended. The constant b(x) is the mean net social cost of punishing the offender. I say "net" social cost because punishing an offender may have multiple cost components, not all of which need be positive. Positive social costs may be incurred for the prosecution of offenses, the incapacitation of convicted offenders, and the deleterious effects of punishment on offenders themselves. Negative social costs may arise to the extent that society views retribution for offenses as a social good. Again, integrating across P(x) gives the aggregate social cost of punishing apprehended offenders.

The third term gives the aggregate cost of performing searches. The positive constant c(x) is the mean cost of performing a search on a person with covariates x. The integral $\int c(x)\cdot z(x)dP(x)$ is the aggregate cost of performing searches.

The planner wants to solve the problem $\min_{z \in Z} S(z)$, where Z is the space of feasible search rules. This minimization problem is separable in x. For each $x \in X$, the optimal search rate for persons with covariates x is

$$(2) \quad z^*(x) \equiv \underset{t \in [0, 1]}{\operatorname{argmin}} \; a(x)\cdot p(t, x)\cdot(1 - t) + b(x)\cdot p(t, x)\cdot t + c(x)\cdot t.$$

Inspection of (2) shows that the planning problem gives multiple reasons for profiling. The cost magnitudes a(x), b(x), or c(x) may vary with x. So may the offense function $p(\cdot, x)$.

The analysis in this paper assumes that $a(x) > b(x) \geq 0$. The assumption that $b(x) \geq 0$ asserts that society does not value retribution so highly as to make punishment a net social good. The assumption that $a(x) > b(x)$ asserts that punishment is less costly to society than the harm caused by completed offenses. These assumptions differ considerably from those of Knowles, Persico, and Todd (2001) and of Persico

(2002). Translating their model of police-initiated profiling into my notation for the social planning problem, they assumed that $a(x) = 0$ and $b(x) < 0$.

Illustration: Linear Deterrence

Manski (2005b) uses the analytically simple case of linear deterrence to illustrate how the optimal search rate may depend on the cost magnitudes and offense function. Let $\rho(x) \equiv p(0, x)$ denote the offense rate for persons with covariates x when their search rate is zero. Search deters offense linearly if $p(t, x) = \rho(x)\cdot(1 - t)$, $t \in [0, 1]$. In terms of the threshold-crossing model, linear deterrence means that a fraction of $1 - \rho(x)$ of persons with covariates x have negative thresholds and, hence, do not commit an offense even when the search rate is zero. The remaining fraction $\rho(x)$ have thresholds distributed uniformly on the interval [0, 1].

In this setting, the optimal search rate for persons with covariates x is

$$(3)\quad z^*(x) = \underset{t \in [0, 1]}{\operatorname{argmin}}\ a(x)\cdot\rho(x)\cdot(1 - t)^2 + b(x)\cdot\rho(x)\cdot t(1 - t) + c(x)\cdot t.$$

The first-order condition for the extremum of the quadratic function in t on the right hand side is

$$\begin{aligned}(4)\quad 0 &= 2a(x)\cdot\rho(x)\cdot t - 2a(x)\cdot\rho(x) + b(x)\cdot\rho(x) - 2b(x)\cdot\rho(x)\cdot t + c(x)\\ &= 2[a(x) - b(x)]\cdot\rho(x)\cdot t - [2a(x) - b(x)]\cdot\rho(x) + c(x).\end{aligned}$$

With $a(x) > b(x)$, this first-order condition implies that the global minimum is at

$$(5)\quad t^*(x) = \frac{[2a(x) - b(x)]\cdot\rho(x) - c(x)}{2[a(x) - b(x)]\cdot\rho(x)}\ .$$

Hence, the optimal search rate is

$$
\begin{aligned}
(6)\quad z^*(x) &= 0 && \text{if } \; t^* < 0, \\
&= t^*(x) && \text{if } \; 0 \le t^* \le 1, \\
&= 1 && \text{if } \; t^* > 1.
\end{aligned}
$$

Holding the cost magnitudes fixed, the optimal search rate $z^*(x)$ increases with $\rho(x)$ and the offense rate implied by optimal search decreases with $\rho(x)$. The optimal search rate is zero if $\rho(x) \le c(x)/[2a(x) - b(x)]$, making the offense rate equal $\rho(x)$. It is optimal to search all persons with covariates x if $\rho(x) \ge c(x)/b(x)$, making their offense rate equal zero. In intermediate cases it is optimal to search with probability $t^*(x)$, which increases with $\rho(x)$. This makes the offense rate equal $[c(x) - b(x)\cdot\rho(x)]/\{2[a(x) - b(x)]\}$, which decreases with $\rho(x)$.

3. Partial Knowledge of Deterrence

3.1. Empirical Evidence and Credible Assumptions

Solution of the planning problem of Section 2 requires essentially complete knowledge of the offense function $p(\cdot, \cdot)$. However, this knowledge generally is unavailable in practice. Empirical evidence and credible assumptions may restrict the form of the offense function, but they rarely if ever pin it down fully.

To demonstrate how a planner with partial knowledge of deterrence may choose a profiling policy, I consider decision making in a particular informational setting. I suppose that the planner observes the offense rates of a study population whose search rule has previously been chosen. He thinks it credible to assume that the study population and the population of interest have the same offense function. He also thinks it credible to assume that the offense rate weakly decreases as the search rate increases. However, he does not know anything about the magnitude of the deterrent effect of search.

Let r(x) denote the search rate applied to persons with covariates x in the study population and let q(x) denote the realized offense rate of these persons. Formally, I maintain

Assumption 1 (Study Population): The planner observes [r(x), q(x)], $x \in X$. The planner knows that q(x) = p[r(x), x], $x \in X$.

Assumption 2(Search Weakly Deters Crime): The planner knows that, for each $x \in X$, p(t, x) is weakly decreasing in t.

Taken together, these assumptions imply that

$$
\begin{aligned}
(7)\quad & t \le r(x) \;\Rightarrow\; p(t, x) \ge q(x), \\
& t \ge r(x) \;\Rightarrow\; p(t, x) \le q(x).
\end{aligned}
$$

Thus, the planner knows that $p(\cdot, x)$ is weakly decreasing and satisfies (7).

Assumptions 1 and 2 may be more realistic than the traditional economic assumption that the planner knows the offense function. A planner often observes the search and offense rates of a study population; in particular, he may observe the past search and offense rates of his own jurisdiction. In this and some other cases, it may be reasonable to suppose that the study population and the population of interest are sufficiently similar as to share (at least approximately) the same offense function.

It usually is reasonable to think that search weakly deters crime. In particular, this holds under the threshold-crossing model cited in Section 2, where $P(t < \tau | x) \le p(t, x) \le P(t \le \tau | x)$. Assumption 2 can hold even if members of the population do not correctly perceive the search rates that they face. It suffices that their perceptions vary monotonically with actual search rates.

Although I conjecture that the informational setting posed here may be realistic, I am aware of no research that describes how law enforcement agencies actually perceive the deterrent effect of search. What is clear is that social scientists have usually found it difficult to learn how policing and sanctions affect offense rates. For example, Blumstein, Cohen, and Nagin (1978) explains why it is so hard to establish the deterrent effect of capital punishment on murder. National Research Council (2001) summarizes the very limited information available on the effectiveness of criminal sanctions in deterring drug supply and use.

3.2. Dominated Search Rules

How should a planner use Assumptions 1 and 2 to choose a profiling policy? Although there is no one "correct" answer to this question, a planner clearly should not choose a search rule that is inferior whatever the actual offense function may be.

Let Γ denote the set of offense functions that are feasible under Assumptions 1 and 2. For $\gamma \in \Gamma$, let

$$(8) \quad S(z, \gamma) \equiv \int a(x)\cdot\gamma[z(x), x]\cdot[1 - z(x)]dP(x) + \int b(x)\cdot\gamma[z(x), x]\cdot z(x)dP(x) + \int c(x)\cdot z(x)dP(x)$$

be the social cost of search rule z when the offense function is γ. Rule z is *strictly dominated* if and only if there exists another search rule $z' \in Z$ such that $S(z, \gamma) > S(z', \gamma)$ for all $\gamma \in \Gamma$.

The planning problem is separable in x, so it suffices to consider each covariate value separately. Suppressing the symbol x to simplify the notation, let $t \in [0, 1]$ and $s \in [0, 1]$ designate feasible choices for the search rate. Let

(9) $d(t, s; \gamma) \equiv [a(1-t) + bt]\gamma(t) + ct - [a(1-s) + bs]\gamma(s) - cs$

be the difference in social cost between application of search rates t and s when the offense function is γ. Let $D(t, s) \equiv \sup_{\gamma \in \Gamma} d(t, s; \gamma)$. Search rate s is strictly dominated if and only if there exists a t such that $d(t, s; \gamma) < 0$ for all $\gamma \in \Gamma$. An easily verifiable sufficient condition for strict dominance is that $D(t, s) < 0$.

Lemma 1 evaluates D(t, s). Then Lemma 2 uses the result to determine a set of dominated search rates. It might be thought that Assumptions 1 and 2 are too weak to yield interesting dominance findings, but Lemma 2 shows that they suffice to eliminate a significant class of search rules. All else equal, small (large) values of s are dominated when the search cost c is small (large).

Lemma 1: Let Assumptions 1 and 2 hold. Then

(i) $t < s \le r \Rightarrow D(t, s) = a(1-t) + bt - aq(1-s) - bqs + c(t-s)$,

(ii) $t < r < s \Rightarrow D(t, s) = a(1-t) + bt + c(t-s)$,

(iii) $s \le t < r \Rightarrow D(t, s) = [c - q(a-b)](t-s)$,

(iv) $r \le t < s \Rightarrow D(t, s) = aq(1-t) + bqt + c(t-s)$,

(v) $r \le s \le t \Rightarrow D(t, s) = c(t-s)$,

(vi) $s < r \le t \Rightarrow D(t, s) = [c - q(a-b)](t-s)$. □

Proof: (i) $t < s \le r \Rightarrow \gamma(t) \ge \gamma(s) \ge q$. Maximization over Γ occurs by setting $\gamma(t) = 1$ and $\gamma(s) = q$. This gives $D(t, s) = a(1-t) + bt + ct - a(1-s)q - bsq - cs$.

(ii) $t < r < s \Rightarrow \gamma(t) \ge q \ge \gamma(s)$. Maximization over Γ occurs by setting $\gamma(t) = 1$ and $\gamma(s) = 0$. This gives $D(t, s) = a(1-t) + bt + ct - cs$.

(iii) $s \le t < r \Rightarrow \gamma(s) \ge \gamma(t) \ge q$. Maximization over Γ occurs by setting $\gamma(t) = \gamma(s) = \delta$ for some $\delta \ge q$. This gives $D(t, s) = [c - \delta(a-b)](t-s)$. Given that $a > b$ and $t \ge s$, the maximum is attained by setting

$\delta = q$. Hence, $D(t, s) = [c - q(a - b)](t - s)$.

(iv) $r \le t < s \Rightarrow q \ge \gamma(t) \ge \gamma(s)$. Maximization over Γ occurs by setting $\gamma(t) = q$ and $\gamma(s) = 0$. This gives $D(t, s) = a(1 - t)q + btq + ct - cs$.

(v) $r \le s \le t \Rightarrow q \ge \gamma(s) \ge \gamma(t)$. Maximization over Γ occurs by setting $\gamma(t) = \gamma(s) = \delta$ for some $\delta \le q$. This gives $D(t, s) = [c - \delta(a - b)](t - s)$. Given that $a > b$ and $t \ge s$, the maximum is attained by setting $\delta = 0$. Hence, $D(t, s) = c(t - s)$

(vi) $s < r \le t \Rightarrow \gamma(s) \ge q \ge \gamma(t)$. Maximization over Γ occurs by setting $\gamma(t) = \gamma(s) = q$. This gives $D(t, s) = [c - q(a - b)](t - s)$.

Q. E. D.

Lemma 2: Let Assumptions 1 and 2 hold. Search rate s is strictly dominated if any of these conditions hold:

(a) Let $c < (a - b)q$. Then s is strictly dominated if $s < r$.

(b) Let $c > (a - b)q$. Then s is strictly dominated if $s > r + [aq(1 - r) + bqr]/c$.

(c) Let $c > a - b$. Then s is strictly dominated if $a(1 - q)/[c - q(a - b)] < s \le r$ or if $s > \max(r, a/c)$. □

Proof: (a) Part (vi) of Lemma 1 showed that if $s < r \le t$, then $D(t, s) = [c - q\cdot(a - b)]\cdot(t - s)$. Hence, $D(t, s) < 0$ for all such (t, s).

(b) Part (iv) of Lemma 1 showed that if $r \le t < s$, then

$$D(t, s) = aq(1 - t) + bqt + c(t - s) = aq + [c - q(a - b)]t - cs.$$

Consider the right hand side as a function of t. The function is minimized at $t = r$, giving $D(r, s) = aq + [c - q(a - b)]r - cs$. If $s > r + [aq(1 - r) + bqr]/c$, then $D(r, s) < 0$.

(c) Part (i) of Lemma 1 showed that if $t < s \le r$, then

$$D(t, s) = a(1 - t) + bt - aq(1 - s) - bqs + c(t - s) = a(1 - q) + [c - (a - b)]t - [c - q(a - b)]s.$$

Consider the right hand side as a function of t. The function is minimized at $t = 0$, giving $D(0, s) = a(1 - q) - [c - q(a - b)]s$. If $s > a(1 - q)/[c - q(a - b)]$, then $D(0, s) < 0$.

Part (ii) of Lemma 1 showed that if $t < r < s$, then

$$D(t, s) = a(1 - t) + bt + c(t - s) = a + [c - (a - b)]t - cs.$$

Consider the right hand side as a function of t. The function is minimized at $t = 0$, giving $D(0, s) = a - cs$. If $s > a/c$, then $D(0, s) < 0$.

Q. E. D.

Elimination of dominated search rules takes one part way toward solution of the planning problem. The literature on decision theory does not provide a consensus prescription for a complete solution, but it does offer various criteria that ensure choice of an undominated alternative. Particularly familiar to economists is Bayesian decision theory which, in the present setting, recommends that the planner place a subjective distribution on $p(\cdot, \cdot)$, say Ψ, and minimize subjective expected social cost. The social cost function (1) is linear in $p(\cdot, \cdot)$, so subjective expected social cost is

$$(10) \quad E_\Psi[S(z)] = \int a(x)\cdot\pi[z(x), x]\cdot[1 - z(x)]dP(x) + \int b(x)\cdot\pi[z(x), x]\cdot z(x)dP(x) + \int c(x)\cdot z(x)dP(x),$$

where $\pi(\cdot, x) \equiv E_\Psi[p(\cdot, x)]$ is the subjective mean of $p(\cdot, x)$. Thus, a Bayesian planner acts as a pseudo-optimizer, using the subjective expected offense function π as if it were the actual offense function p.

The Bayesian prescription may be sensible if a planner can substantiate his choice of π, but pseudooptimization has no special appeal otherwise. A planner who does not want to go the Bayesian route can reasonably apply the minimax or minimax-regret criterion to choose a search rule. These are general principles using whatever partial knowledge of the offense function the planner may have. Each criterion chooses a rule that, in one sense or another, performs uniformly well across all feasible offense functions.

Sections 3.3 and 3.4 describe the minimax and minimax-regret search rules under Assumptions 1 and 2. See Berger (1985) for exposition of the minimax and minimax-regret criteria. See Manski (2004, 2005a) for other applications of these criteria to problems of social planning with partial knowledge of policy impacts.

3.3. Minimax Search

The minimax criterion is

$$(11) \quad \min_{z \in Z} \ \max_{\gamma \in \Gamma} S(z, \gamma).$$

The outer minimization problem is separable in x. Hence, the minimax search rate for persons with covariates x is

$$(12) \quad z^m(x) \equiv \operatorname*{argmin}_{t \in [0, 1]} \ \max_{\gamma \in \Gamma} \ [a(x)\cdot(1 - t) + b(x)\cdot t]\cdot\gamma(t, x) + c(x)\cdot t.$$

Lemma 3 derives the search rule that solves this problem. We find that the minimax search rate for persons with covariates x can take one of three values: 0, r(x), or 1. All else equal, the search rate weakly increases with the cost magnitude a(x) and decreases with b(x) and c(x). It weakly increases with the realized offense rate q(x) if c(x) < a(x) − b(x) and decreases with q(x) otherwise. In what follows, I suppress the symbol x to simplify the notation.

Lemma 3: Under Assumptions 1 and 2, the minimax search rate is

$$(13)\quad z^m = 0 \quad \text{if} \quad c \geq a - b \text{ and } a \leq aq(1-r) + bqr + cr,$$

$$= r \quad \text{if} \quad c \geq a - b \text{ and } a \geq aq(1-r) + bqr + cr$$

$$\text{or if} \quad (a-b)q \leq c < a - b,$$

$$= 1 \quad \text{if} \quad c \leq (a-b)q. \qquad \square$$

Proof: For each value of t, the inner maximization problem in (12) is solved by setting the offense rate to its largest feasible value; that is, $\gamma(t) = 1[t < r] + q \cdot 1[t \geq r]$. Hence,

$$(14)\quad z^m \equiv \underset{t \in [0,\, 1]}{\operatorname{argmin}} \quad [a(1-t) + bt] \cdot \{1[t < r] + q \cdot 1[t \geq r]\} + ct.$$

To solve problem (14), I first consider the two domains $t < r$ and $t \geq r$ separately, and then combine them.

(i) Consider $t < r$. In this domain, the minimization problem is $\min_{t < r}\ a(1-t) + bt + ct$. If $c \geq a - b$, the solution is $t = 0$ and the minimax value is a. If $c < a - b$, the criterion function decreases as $t \to r$, with limit value $a(1-r) + br + cr$.

(ii). Consider $t \geq r$. In this domain, the minimization problem is $\min_{t \geq r}\ a(1-t)q + bqt + ct$. If $c \geq (a-b)q$, the solution is $t = r$ and the minimax value is $a(1-r)q + bqr + cr$. If $c < (a-b)q$, the solution is $t = 1$ and the minimax value is $bq + c$.

Now combine the two domains. If $c \geq a - b$, the solution over $t \in [0, 1]$ is $t = 0$ if $a \leq a(1-r)q + bqr + cr$

and is $t = r$ if $a \geq a(1 - r)q + bqr + cr$. If $(a - b)q \leq c < a - b$, the solution is $t = r$. If $c < (a - b)q$, the solution is $t = 1$.

Q. E. D.

3.3. Minimax-Regret Search

For $\gamma \in \Gamma$, let $S^*(\gamma) \equiv \min_{z \in Z} S(z, \gamma)$ be the lowest social cost achievable by any feasible search rule when the offense function is γ. The regret of search rule z in state of nature γ is $S(z, \gamma) - S^*(\gamma)$. The minimax-regret criterion is

$$(15) \quad \min_{z \in Z} \sup_{\gamma \in \Gamma} S(z, \gamma) - S^*(\gamma).$$

The outer minimization problem is separable in x. Hence, the minimax-regret search rate for persons with covariates x is

$$(16) \quad z^{mr}(x)$$

$$= \underset{t \in [0, 1]}{\operatorname{argmin}} \sup_{\gamma \in \Gamma} \Big\{ [a(x)\cdot(1 - t) + b(x)\cdot t]\cdot\gamma(t, x) + c(x)\cdot t \; - \min_{s \in [0, 1]} [a(x)\cdot(1 - s) + b(x)\cdot s]\cdot\gamma(s, x) + c(x)\cdot s] \Big\}$$

$$= \underset{t \in [0, 1]}{\operatorname{argmin}} \sup_{\gamma \in \Gamma,\, s \in [0, 1]} [a(x)\cdot(1 - t) + b(x)\cdot t]\cdot\gamma(t, x) + c(x)\cdot t \; - \; [a(x)\cdot(1 - s) + b(x)\cdot s]\cdot\gamma(s, x) - c(x)\cdot s$$

$$= \underset{t \in [0, 1]}{\operatorname{argmin}} \max_{s \in [0, 1]} D(t, s; x),$$

where D(t, s; x) was defined in Section 3.2, with the notation x suppressed. Derivation of an explicit expression for the minimax-regret search rate is not intrinsically difficult, but it is laborious to catalog how the rate depends on the cost magnitudes and the evidence observed in the study population. Lemma 4 performs the derivation for the special case where $b = 0$ and $c < aq$. In what follows, the notation x is again suppressed.

Lemma 4: Let $b = 0$ and $c < aq$. Under Assumptions 1 and 2, the minimax-regret search rate is

$$(17) \quad z^{mr} = (aq + cr)/(aq + c). \qquad \square$$

Proof: With $b = 0$, the values of $D(t, s)$ derived in Lemma 1 become

$t < s \le r \Rightarrow D(t, s) = a(1 - t) - aq(1 - s) + c(t - s),$

$t < r < s \Rightarrow D(t, s) = a(1 - t) + c(t - s),$

$s \le t < r \Rightarrow D(t, s) = (c - aq)(t - s),$

$r \le t < s \Rightarrow D(t, s) = aq(1 - t) + c(t - s),$

$r \le s \le t \Rightarrow D(t, s) = c(t - s),$

$s < r \le t \Rightarrow D(t, s) = (c - aq)(t - s).$

Consider $t < r$. I first fix t and maximize $D(t, s)$ over $s \in [0, 1]$. There are three cases to consider:

(i) Given that $c < aq$, $\max_{s: t < s \le r} D(t, s)$ occurs at $s = r$, so $\max_{s: t < s \le r} D(t, s) = a(1 - t) - aq(1 - r) + c(t - r)$.

(ii) $\sup_{s: t < r < s} D(t, s)$ occurs at $s = r$, so $\sup_{s: t < r < s} D(t, s) = a(1 - t) + c(t - r)$.

(iii) $\max_{s \le t < r} D(t, s)$ occurs at $s = t$, so $\max_{s \le t < r} D(t, s) = 0$.

The supremum in case (ii) exceeds the maxima in cases (i) and (iii). Hence, $\sup_{s \in [0, 1]} D(t, s) = a(1 - t) + ct - cr$.

Minimization over $t < r$ of the expression $a(1 - t) + ct - cr$ yields the minimax-regret search rate within this restricted range of search rates. Given that $c < a$, it follows that $a(1 - t) + ct - cr$ decreases with t. Hence, $\inf_{t < r} \sup_{s \in [0, 1]} D(t, s) = a(1 - r)$.

Now consider $t \geq r$. Again, I first fix t and maximize $D(t, s)$ over $s \in [0, 1]$. There are three cases to consider:

(i) $\sup_{s: r \leq t < s} D(t, s)$ occurs at $s = t$, so $\sup_{s: r \leq t \leq s} D(t, s) = aq(1 - t)$.

(ii) $\max_{s: r \leq s \leq t} D(t, s)$ occurs at $s = r$, so $\max_{s: r \leq s \leq t} D(t, s) = c(t - r)$.

(iii) $\max_{s: s < r \leq t} D(t, s)$ occurs at $s = 0$, so $\max_{s: s < r \leq t} D(t, s) = (c - aq)t$.

The supremum in case (iii) is non-positive. Hence, $\sup_{s \in [0, 1]} D(t, s) = \sup[aq(1 - t), c(t - r)]$.

Minimization over $t \geq r$ of $\sup[aq(1 - t), c(t - r)]$ yields the minimax-regret search rate within this restricted range of search rates. The expression $aq(1 - t)$ falls from $aq(1 - r)$ to 0 as t rises from r to 1. The expression $c(t - r)$ rises from 0 to $c(1 - r)$ as t rises from r to 1. Hence, $\sup[aq(1 - t), c(t - r)]$ is minimized when t solves the equation $aq(1 - t) = c(t - r)$; that is, when $t = (aq + cr)/(aq + c)$. Hence, $\min_{t \geq r} \sup_{s \in [0, 1]} D(t, s) = caq(1 - r)/(c + aq)$.

Finally, compare the minimax-regret values over the two ranges $t < r$ and $t \geq r$. The latter value is smaller than the former one. Hence, $(aq + cr)/(aq + c)$ is the overall minimax-regret search rate.

Q. E. D.

4. Ex Post Search

When search is ex post, the social cost function is given not by S(z) but rather by

(18) $$S'(z) = \int a(x)\cdot p[z(x), x]dP(x) + \int b(x)\cdot p[z(x), x]\cdot z(x)dP(x) + \int c(x)\cdot z(x)dP(x).$$

The difference between $S'(z)$ and $S(z)$ is that the probability of an offense causing social harm is now $p[z(x), x]$, whereas earlier it was $p[z(x), x]\cdot[1 - z(x)]$.

Illustration: Linear Deterrence

Suppose that ex post search has a linear deterrent effect, so $p(t, x) = \rho(x)\cdot (1 - t)$. Then the optimal search rate for persons with covariates x is

$$(19)\quad z'(x) = \underset{t \in [0,1]}{\operatorname{argmin}}\ a(x)\cdot\rho(x)\cdot(1 - t) + b(x)\cdot\rho(x)\cdot t(1 - t) + c(x)\cdot t.$$

The first-order condition for the extremum of the quadratic function in t on the right hand side is

$$(20)\quad 0 = - a(x)\cdot\rho(x) + b(x)\cdot\rho(x) - 2b(x)\cdot\rho(x)\cdot t + c(x).$$

This first-order condition gives a global maximum at

$$(21)\quad t'(x) = \tfrac{1}{2} + \frac{c(x) - a(x)\cdot\rho(x)}{2b(x)\cdot\rho(x)} .$$

Hence, the optimal search rate must be zero or one. Setting $t = 0$ yields social cost $a(x)\cdot\rho(x)$ and setting $t = 1$ yields social cost $c(x)$. Hence,

$$(22)\quad \begin{aligned} z'(x) &= 0 \qquad \text{if } c(x) \geq a(x)\cdot\rho(x), \\ &= 1 \qquad \text{if } c(x) \leq a(x)\cdot\rho(x). \end{aligned}$$

Holding the parameter values $\{a(x), b(x), c(x), \rho(x)\}$ fixed, it is of interest to compare the optimal search rates with ex ante and ex post search, given in (6) and (22) respectively. We find that $z^*(x) = 0 \Rightarrow z'(x) = 0$ and $z^*(x) = 1 \Rightarrow z'(x) = 1$. When $0 < z^*(x) < 1$, search rate $z'(x)$ equals zero or one, depending on the parameter values. A perhaps surprising difference between $z^*(x)$ and $z'(x)$ is that the former depends on the cost magnitude $b(x)$ but the latter does not.

4.1. Dominated Search Rules

As in Section 3, the planning problem is separable in x, so it suffices to consider each covariate value separately. Suppressing the symbol x to simplify the notation, let $t \in [0, 1]$ and $s \in [0, 1]$ designate feasible choices for the search rate. Let

$$(23) \quad d'(t, s; \gamma) \equiv (a + bt)\gamma(t) + ct - (a + bs)\gamma(s) - cs$$

be the difference in social cost between application of search rates t and s when the offense function is γ. Let $D'(t, s) \equiv \sup_{\gamma \in \Gamma} d'(t, s; \gamma)$. A sufficient condition for search rate s to be strictly dominated is that there exists a t such that $D'(t, s) < 0$.

Lemma 5 evaluates $D'(t, s)$. Lemma 6 uses the result to determine dominated search rates.

Lemma 5: Let S′ be the social cost function. Let Assumptions 1 and 2 hold. Then

(i) $t < s \le r \Rightarrow D'(t, s) = a + bt - aq - bqs + c(t - s)$,

(ii) $t < r < s \Rightarrow D'(t, s) = a + bt + c(t - s)$,

(iii) $s \le t < r \Rightarrow D'(t, s) = (b + c)(t - s)$,

(iv) $r \le t < s \Rightarrow D'(t, s) = aq + bqt + c(t - s)$,

(v) $r \le s \le t \Rightarrow D'(t, s) = (bq + c)(t - s)$,

(vi) $s < r \le t \;\Rightarrow\; D'(t, s) = (bq + c)(t - s)$. □

Proof: There are six distinct cases, as follows:

(i) Let $t < s \le r$. Then $\gamma(t) \ge \gamma(s) \ge q$. Maximization over Γ occurs by setting $\gamma(t) = 1$ and $\gamma(s) = q$. This gives $D'(t, s) = a + bt + ct - aq - bsq - cs$.

(ii) Let $t < r < s$. Then $\gamma(t) \ge q \ge \gamma(s)$. Maximization over Γ occurs by setting $\gamma(t) = 1$ and $\gamma(s) = 0$. This gives $D'(t, s) = a + bt + ct - cs$.

(iii) Let $s \le t < r$. Then $\gamma(s) \ge \gamma(t) \ge q$. Maximization over Γ occurs by setting $\gamma(t) = \gamma(s) = \delta$ for some $\delta \ge q$. This gives $D'(t, s) = (b\delta + c)(t - s)$. Given that $t \ge s$, the maximum is attained by setting $\delta = 1$. This gives $D'(t, s) = (b + c)(t - s)$.

(iv) Let $r \le t < s$. Then $q \ge \gamma(t) \ge \gamma(s)$. Maximization over Γ occurs by setting $\gamma(t) = q$ and $\gamma(s) = 0$. This gives $D'(t, s) = aq + btq + ct - cs$.

(v) Let $r \le s \le t$. Then $q \ge \gamma(s) \ge \gamma(t)$. Maximization over Γ occurs by setting $\gamma(t) = \gamma(s) = \delta$ for some $\delta \le q$. Given that $t \ge s$, the maximum is attained by setting $\delta = q$. This gives $D'(t, s) = (bq + c)(t - s)$.

(vi) Let $s < r \le t$. Then $\gamma(s) \ge q \ge \gamma(t)$. Maximization over Γ occurs by setting $\gamma(t) = \gamma(s) = q$. This gives $D'(t, s) = (bq + c)(t - s)$.

Q. E. D.

Lemma 6: Let S' be the social cost function. Let Assumptions 1 and 2 hold. Search rate s is strictly dominated if any of these conditions hold: (a) $a(1 - q)/(c + bq) < s \le r$; (b) $s > \max(r, a/c)$; (c) $s > r + (aq + bqr)/c$. □

Proof: (a) Part (i) of Lemma 5 showed that if $t < s \le r$, then

$$D'(t, s) = a + bt - aq - bqs + c(t - s) = a(1 - q) + (c + b)t - (c + bq)s.$$

Consider the right hand side as a function of t. The function is minimized at $t = 0$, giving $D'(0, s) = a(1 - q) - (c + bq)s$. If $s > a(1 - q)/(c + bq)$, then $D'(0, s) < 0$.

(b) Part (ii) of Lemma 5 showed that if $t < r < s$, then

$$D'(t, s) = a + bt + c(t - s) = a + (c + b)t - cs.$$

Consider the right hand side as a function of t. The function is minimized at $t = 0$, giving $D'(0, s) = a - cs$. If $s > a/c$, then $D'(0, s) < 0$.

(c) Part (iv) of Lemma 5 showed that if $r \le t < s$, then

$$D'(t, s) = aq + bqt + c(t - s) = aq + (c + bq)t - cs.$$

Consider the right hand side as a function of t. The function is minimized at $t = r$, giving $D'(r, s) = aq + cr + bqr - cs$. If $s > r + (aq + bqr)/c$, then $D'(r, s) < 0$.

Q. E. D.

4.2. Minimax Search

Lemma 7 repeats the derivation of Lemma 3, using S′ rather than S as the social cost function. Whereas the minimax search rate for ex ante search could take the values $\{0, r, 1\}$, we find that it now can take only the values $\{0, r\}$. Moreover, the present minimax search rate is zero whenever the earlier minimax search rate is zero.

Lemma 7: Let S′ be the social cost function. Under Assumptions 1 and 2, the minimax search rate is

$$\text{(24)}\quad z^{m'} = 0 \qquad \text{if} \quad a \le aq + bqr + cr,$$
$$= r \qquad \text{if} \quad a \ge aq + bqr + cr. \qquad \square$$

Proof: The minimax search rate is

$$\text{(25)}\quad z^{m'} \equiv \underset{t \in [0,\,1]}{\text{argmin}} \ \underset{\gamma \in \Gamma}{\max} \ (a + bt)\gamma(t) + ct.$$

The inner maximization problem is solved by setting the offense rate to $\gamma(t) = 1[t < r] + q\cdot 1[t \ge r]$. Hence,

$$\text{(26)}\quad z^{m'} \equiv \underset{t \in [0,\,1]}{\text{argmin}} \ (a + bt)\cdot\{1[t < r] + q\cdot 1[t \ge r]\} + ct.$$

To solve problem (26), I consider the two domains $t < r$ and $t \ge r$ separately and then combine them.

(i) Let $t < r$. The minimization problem is $\min_{t < r} a + bt + ct$. The solution is $t = 0$. The minimax value is a.

(ii). Let $t \ge r$. The minimization problem is $\min_{t \ge r} aq + bqt + ct$. The solution is $t = r$. The minimax value is $aq + bqr + cr$.

Hence, the solution over $t \in [0, 1]$ is $t = 0$ if $a \le aq + bqr + cr$ and is $t = r$ if $a \ge aq + bqr + cr$.

Q. E. D.

4.3. Minimax-Regret Search

Lemma 8 repeats the derivation of Lemma 4 using S' rather than S as the social cost function. Whereas the minimax-regret search rate for ex ante search in the setting of Lemma 4 was a unique value between r and one, we find that in the corresponding setting of Lemma 8 it equals zero in some cases and takes any value between r and one in others.

Lemma 8: Let S′ be the social cost function. Let b = 0 and c < aq. Under Assumptions 1 and 2, the minimax-regret search rate is

$$(27)\quad z^{mr\prime} = 0 \qquad \text{if} \quad a \le aq + cr,$$
$$= \text{all } t \ge r \qquad \text{if} \quad a > aq + cr. \qquad \square$$

Proof: With b = 0, the quantity $D'(t, s)$ defined in Lemma 5 becomes

$t < s \le r \Rightarrow D'(t, s) = a(1 - q) + c(t - s),$

$t < r < s \Rightarrow D'(t, s) = a + c(t - s),$

$s \le t < r \Rightarrow D'(t, s) = c(t - s),$

$r \le t < s \Rightarrow D'(t, s) = aq + c(t - s),$

$r \le s \le t \Rightarrow D'(t, s) = c(t - s),$

$s < r \le t \Rightarrow D'(t, s) = c(t - s).$

Consider t < r. I first fix t and maximize $D'(t, s)$ over $s \in [0, 1]$. There are three cases to consider:

(i) $\sup_{s:\, t < s \le r} D'(t, s)$ occurs at s = t as s → t, so $\sup_{s:\, t < s \le r} D'(t, s) = a(1 - q)$.

(ii) $\sup_{s:\, t < r < s} D'(t, s)$ occurs at s = r as s → r, so $\sup_{s:\, t < r < s} D'(t, s) = a + c(t - r)$.

(iii) $\max_{s \le t < r} D'(t, s)$ occurs at s = 0, so $\max_{s \le t < r} D'(t, s) = ct$.

Given that c < aq, the supremum in case (ii) exceeds those in cases (i) and (iii). Hence, $\max_{s \in [0, 1]} D'(t, s) =$

$a + c(t - r)$.

Minimization over $t < r$ of the expression $a + c(t - r)$ yields the minimax-regret search rate within this restricted range of search rates. The minimum occurs at $t = 0$, yielding $\min_{t < r} \max_{s \in [0, 1]} D'(t, s) = a - cr$.

Now consider $t \geq r$. Again fix t and maximize $D'(t, s)$ over $s \in [0, 1]$. There are three cases to consider:

(i) $\sup_{s: r \leq t < s} D'(t, s)$ occurs at $s = t$ as $s \rightarrow t$, so $\sup_{s: r \leq t \leq s} D'(t, s) = aq$.

(ii) $\max_{s: r \leq s \leq t} D'(t, s)$ occurs at $s = r$, so $\max_{s: r \leq s \leq t} D'(t, s) = c(t - r)$.

(iii) $\max_{s: s < t \leq r} D'(t, s)$ occurs at $s = 0$, so $\sup_{s: s < r \leq t} D'(t, s) = ct$.

Given that $c < aq$, the supremum in case (i) exceeds those in cases (ii) and (iii). Hence, $\sup_{s \in [0, 1]} D'(t, s) = aq$ for all $t \geq r$.

Finally, compare the minimax-regret values over the two ranges $t < r$ and $t \geq r$. If $a - cr \leq aq$, then the minimax-regret search rate is $t = 0$. If $a - cr > aq$, then all $t \geq r$ minimize maximum regret.

Q. E. D.

5. Variations on the Planning Problem

This paper has shown how a social planner having partial knowledge of population offense behavior can reasonably choose a search profiling policy. I say "reasonably" rather than "optimally" choose because our planner does not have enough information to solve the optimization problem that he would like to solve. What he can do is eliminate dominated policies and use some well-defined criterion to choose among the undominated policies. Minimax and minimax-regret are two such criteria, although they certainly are not the only ones.

The informational setting posed in Assumptions 1 and 2 more realistic than the traditional economic assumption that planners know how policy affects population behavior. However, I can only conjecture that these assumptions reasonably approximate what law enforcement agencies actually know about the deterrent effect of search. Fresh analysis would be required for other informational settings, but the basic message of the paper would remain applicable. A planner can use whatever knowledge he has to eliminate dominated search rules and then use minimax, minimax-regret, or another criterion to choose an undominated rule.

Many structural variations on the planning problem warrant attention. It may be that search only sometimes apprehends offenders, rather than always as assumed here. A planner may be able to implement both ex ante and ex post search rules, rather than one or the other as assumed here. A planner may be able to choose not only a search rule but also the severity with which apprehended offenders are punished.

Offender behavior may differ from the assumptions maintained here. Offense decisions may have endogenous social interactions, each person's decision to commit an offense depending not only on the search rate that he faces but also on the prevalence of offenses within the population. It may be that the personal covariates that planners observe are malleable. When profiling makes search rates vary with malleable covariates, persons may choose to manipulate their covariate values to lower the probability of search.

Researchers contemplating analysis of endogenous social interactions and malleable covariates should be aware that these phenomena substantially complicate the determination of optimal, dominated, minimax, and minimax-regret search rules. Throughout this paper, the planning problem was separable across persons with different covariates, a fact that enormously simplified analysis. Endogenous social interactions and malleable covariates make the planning problem non-separable; hence, much more difficult to study.

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